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Hybrid quantum computation based on repeat-until-success scheme

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Abstract

In this paper, we focus on a hybrid quantum computing architecture using stationary qubits inside an optical cavity and flying qubits (photons). It has been shown that direct qubit–qubit interactions for two-qubit gate implementations can be replaced by the experimentally less demanding generation of single photons on demand and linear optics photon pair measurements. The outcomes of these measurements indicate either the completion of the gate or the presence of the original qubits such that the operation can be repeated until success.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

The polarization (or temporal) degrees of freedom of photons have often been preferred as a representation of qubits due to their long decoherence times and high speed as well as their ability to distribute themselves in an optical network. However, photons cannot interact directly with each other. Without nonlinear effects, photons can only be entangled to each other via post-selected measurements and local operations. Moreover, linear optics alone does not permit complete Bell measurements [1]. Thus such entangling operations between photons are necessarily probabilistic. Moreover, in order to achieve success probabilities close to unity, one requires the presence of highly entangled ancilla states and quantum teleportation [2] as a universal quantum primitive [3].

Since the seminal proposal by Knill *et al* [2], substantial work has been done to reduce the required resources needed for a realization of linear optics quantum computing [4]. In fact, there have already been some preliminary feasibility studies on linear optics quantum computing [5].

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Photons can be transmitted easily from point to point. Hence they are often regarded as 'flying' qubits. However, there is a trade-off between this feature and the ease of storage: it is generally difficult to store them and to use them as a quantum memory or 'stationary' qubits. On the other hand, qubits realized through atoms and ions provide good quantum memory due to the relatively long decoherence times of their internal ground states. For stationary qubits, it is relatively easy to implement single qubit rotations and read out information with a very high precision. Experiments done in Innsbruck and Boulder have already demonstrated the feasibility of such two-qubit gates for ion trap quantum computing [6, 7]. However, these two-qubit gate operations are in general relatively vulnerable to decoherence and ion trap quantum computing with many qubits is still a challenge.

It is therefore natural to consider a hybrid platform based on both flying and stationary qubits. Such schemes have been explored [8–10], exploiting the benefits of stationary and flying qubits and encoding the logical qubit in the ground states of a single atom as well as in the polarization state of a photon. Recently, we have proposed a repeat-until-success (RUS) quantum computing scheme [11, 12] that is robust, scalable and offers the possibility of storing qubits for a long time.

In section 2, we describe the basic idea in the RUS scheme based on the single atom cavity system. We also note that we could, in principle, construct cluster state computing through a probabilistic scheme. One such example is based on dipole induced transparency and this scheme is briefly sketched in section 3. In section 4, we briefly consider inefficient detectors and its implications in the RUS computing scheme. In section 5, we summarize the main ideas in this talk.

2. Repeat-until-success scheme

Consider two stationary qubits (for instance two single-atom cavities) [11]. Suppose the two stationary qubits are initially prepared in a linear superposition of two internal states $|g\rangle$ and $|m\rangle$,

$$|\psi_{\text{in}}\rangle = \tfrac{1}{2}(|g\rangle + |m\rangle)(|g\rangle + |m\rangle). \tag{1}$$

Suppose that a photon is generated in each of the two sources, whose state (i.e. its polarization or generation time) depends on the state of the source. Thus if the state of the source is $|g\rangle$, then the corresponding photon created is $|h\rangle$, and if the source is prepared in state $|m\rangle$ then the corresponding photon is $|v\rangle$. This operation can be interpreted as an entangling gate between the state of the atom and the state of the photon created. Thus a simultaneous creation of a photon in each source can then transfer the initial state (1) into

$$\begin{aligned}|\psi_{\text{enc}}\rangle &= \tfrac{1}{2}(|g\rangle|h\rangle + |m\rangle|v\rangle)(|g\rangle|h\rangle + |m\rangle|v\rangle)\\ &= \tfrac{1}{2}(|gg\rangle|hh\rangle + |gm\rangle|hv\rangle + |mg\rangle|vh\rangle + |mm\rangle|vv\rangle).\end{aligned} \tag{2}$$

Once the photons have been created, an entangling phase gate (for the photons) can be implemented by allowing each photon to go through a Hadamard polarizer, resulting in the state

$$\begin{aligned}|\psi'_{\text{enc}}\rangle = \tfrac{1}{2}(&|gg\rangle(|hh\rangle + |hv\rangle + |vh\rangle + |vv\rangle) + |gm\rangle(|hh\rangle - |hv\rangle + |vh\rangle - |vv\rangle)\\ &+ |mg\rangle(|hh\rangle + |hv\rangle - |vh\rangle - |vv\rangle) + |mm\rangle(|hh\rangle - |hv\rangle - |vh\rangle + |vv\rangle)).\end{aligned} \tag{3}$$

The photons are then directed into a polarizing beam splitter (PBS) which transmits photons with state $|h\rangle$ but reflects photons with state $|v\rangle$. Thus photons with the state $|hh\rangle$ and $|vv\rangle$

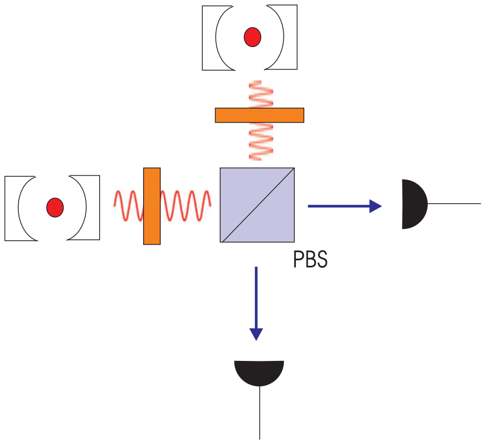

Figure 1. Schematic set-up for the detection of photons emitted from the single-photon source. The photons first go through Hadarmard polarizers followed by a polarizing beam-splitter.

maintain separate paths after the PBS (i.e. anti-bunch) while photons with state $|hv\rangle$ and $|vh\rangle$ bunch. The schematic set-up for the detection is shown in figure 1.

It is instructive to see that the state after the PBS can be written as

$$|\psi''_{\text{enc}}\rangle = \tfrac{1}{2}\{(|gg\rangle + |mm\rangle)(|hh\rangle + |vv\rangle) + (|gm\rangle + |mg\rangle)(|hh\rangle - |vv\rangle) + (|g\rangle + |m\rangle)(|g\rangle - |m\rangle)|hv\rangle + (|g\rangle - |m\rangle)(|g\rangle + |m\rangle)|vh\rangle\}. \tag{4}$$

With linear optics, it is well known that it is impossible to perform a complete 'Bell state' measurement [1]. Instead, we consider the following states:

$$|\Phi_1\rangle \equiv \tfrac{1}{\sqrt{2}}[|hh\rangle + |vv\rangle], \qquad |\Phi_2\rangle \equiv \tfrac{1}{\sqrt{2}}[|hh\rangle - |vv\rangle],$$
$$|\Phi_3\rangle \equiv |hv\rangle, \qquad |\Phi_4\rangle \equiv |vh\rangle. \tag{5}$$

This can be done easily by placing two additional beam splitters at each output. Thus, one finds that if the outcome is $|\Phi_1\rangle$ or $|\Phi_2\rangle$ (with probability 1/2), the stationary qubit is maximally entangled and if the outcome is $|\Phi_3\rangle$ or $|\Phi_4\rangle$, the atoms can be reset to their original state by appropriate local operations.

For pedagogical reasons, we have restricted the state of the atoms to be an equal superposition of the states $|g\rangle$ and $|m\rangle$. More generally, we could consider the states of the atoms as $|\phi_{\text{in}}\rangle = \alpha_{00}|gg\rangle + \alpha_{01}|gm\rangle + \alpha_{10}|mg\rangle + \alpha_{11}|mm\rangle$, where α_{ij} are arbitrary coefficients of the state of the atoms. On emission of the photons, we get the encoded state $|\phi_{\text{enc}}\rangle = \alpha_{00}|gg\rangle|hh\rangle + \alpha_{01}|gm\rangle|hv\rangle + \alpha_{10}|mg\rangle|vh\rangle + \alpha_{11}|mm\rangle|vv\rangle$.

The entire process of allowing the photons to go through polarizers and polarizing beam splitters followed by appropriate measurements in partial Bell basis corresponds to a generic measurement in some mutually unbiased basis. In this case, we consider the basis

$$|\Phi_1\rangle = \tfrac{1}{\sqrt{2}}(|x_1x_2\rangle + |y_1y_2\rangle) \qquad |\Phi_2\rangle = \tfrac{1}{\sqrt{2}}(|x_1x_2\rangle - |y_1y_2\rangle)$$
$$|\Phi_3\rangle = |x_1y_2\rangle \qquad |\Phi_3\rangle = |x_2y_1\rangle, \tag{6}$$

where $|x_1\rangle = \frac{1}{\sqrt{2}}(|h\rangle + |v\rangle), |x_2\rangle = \frac{\mathrm{i}}{\sqrt{2}}(|h\rangle - |v\rangle), |y_1\rangle = \frac{1}{\sqrt{2}}(|h\rangle - |v\rangle)$ and $|y_2\rangle = \frac{1}{\sqrt{2}}(|h\rangle + |v\rangle) = |x_1\rangle$. Expanding Φ_i $(i = 1, \ldots, 4)$ in terms of $|h\rangle$ and $|v\rangle$ gives

$$\begin{aligned}
|\Phi_1\rangle &= \frac{\exp(\mathrm{i}\pi/4)}{2}(|hh\rangle - \mathrm{i}|hv\rangle + \mathrm{i}|vh\rangle - |vv\rangle)\\
|\Phi_2\rangle &= \frac{-\exp(-\mathrm{i}\pi/4)}{2}(|hh\rangle + \mathrm{i}|hv\rangle - \mathrm{i}|vh\rangle - |vv\rangle)\\
|\Phi_3\rangle &= \frac{1}{2}(|hh\rangle + |hv\rangle + |vh\rangle + |vv\rangle)\\
|\Phi_3\rangle &= \frac{\mathrm{i}}{2}(|hh\rangle - |hv\rangle - |vh\rangle + |vv\rangle).
\end{aligned} \tag{7}$$

Thus, it is easily seen that a measurement in the designated basis yields with equal probability the following results:

$$\begin{aligned}
|\psi_1\rangle &= \exp\left(-\frac{\mathrm{i}\pi}{4}\right) Z_1\left(\frac{\pi}{2}\right) Z_2\left(-\frac{\pi}{2}\right) U_{\mathrm{CZ}}|\psi_{\mathrm{in}}\rangle\\
|\psi_2\rangle &= -\exp\left(\frac{\mathrm{i}\pi}{4}\right) Z_1\left(-\frac{\pi}{2}\right) Z_2\left(\frac{\pi}{2}\right) U_{\mathrm{CZ}}|\psi_{\mathrm{in}}\rangle\\
|\psi_3\rangle &= |\psi_{\mathrm{in}}\rangle\\
|\psi_4\rangle &= -\mathrm{i}Z_1(\pi)Z_2(\pi)|\psi_{\mathrm{in}}\rangle,
\end{aligned} \tag{8}$$

so that clearly a measurement result in $|\Phi_1\rangle$ or $|\Phi_2\rangle$ yields an entangling gate (actually a control-phase gate), namely $|\psi_1\rangle|\psi_2\rangle$, on the qubits, whereas a measurement result in $|\Phi_3\rangle$ or $|\Phi_4\rangle$ yields the original input state ($|\psi_3\rangle$ or $|\psi_4\rangle$), up to some local unitary transformation. Here $Z_i(\theta)$ is the unitary transformation $\mathrm{diag}(1, \exp(-\mathrm{i}\theta))$ acting on the ith atom and $U_{\mathrm{CZ}} = \mathrm{diag}(1, 1, 1, -1)$. Here we have mapped $|h\rangle$ to logical $|0\rangle$ in the usual computational basis and $|v\rangle$ to logical $|1\rangle$.

3. Probabilistic schemes

It is probably interesting to note that for the construction of cluster states it is not essential to have a 'repeat-until-success' scheme. There are several schemes [12–14]. However, in one recent elegant proposal [14], atoms in a cavity interact with a waveguide system and get entangled using dipole induced transparency. Although the atoms at the end of the operations are always entangled, one could in principle repeat the scheme by preparing the atoms again. In the scheme, one considers a cavity containing a single dipole emitter evanescently coupled to two waveguides as in figure 2. The cavity may be detuned from cavity resonance, ω_0 with the vacuum Rabi frequency of the dipole being $\tilde{g}$. Each dipole is assumed to have three relevant states: a ground state $|g\rangle$, an excited state $|e\rangle$ and a long-lived metastable state $|m\rangle$. The transition from the ground state to the excited state is assumed to be resonant with the cavity and the transition from the metastable state to the excited state is off resonance. It can be shown that when the dipole is in state $|m\rangle$, it does not couple to the cavity. However, if the dipole is in state $|g\rangle$, the waveguide transmits perfectly. Thus, we see that the system transforms accordingly as $\hat{a}^\dagger_{\mathrm{in}}|g\rangle|0\rangle \rightarrow \hat{a}^\dagger_{\mathrm{out}}|g\rangle|0\rangle$ and $\hat{a}^\dagger_{\mathrm{in}}|m\rangle|0\rangle \rightarrow -\hat{c}^\dagger_{\mathrm{out}}|m\rangle|0\rangle$.

The entangling scheme essentially uses two Mach–Zehnder interferometers set-up to entangle the atoms in the cavity as shown in figure 3. A weak coherent pulse is then split by a beam splitter and sent to two independent cavities containing dipoles coupled to the waveguides. The dipoles are initialized to $(|g\rangle + |m\rangle)/\sqrt{2}$. The waveguide fields are then mixed on a beam splitter so that either detector A and C register constructive interference.

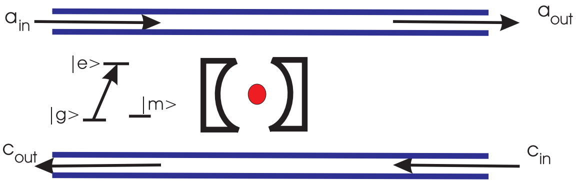

Figure 2. A drop-filter cavity-waveguide system in which a single dipole emitter is evanescently coupled to two waveguides.

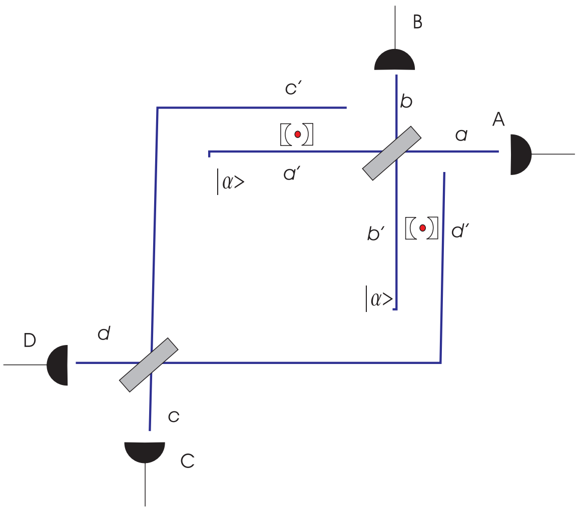

Figure 3. The system is set up so that a detection event in B or D signifies that the atoms have collapsed to the state $1/\sqrt{2}(|gm\rangle - |mg\rangle)$, otherwise the atom are not maximally entangled and the system can be repeated if the atom is reset to a linear superposed state of $|g\rangle$ and $|m\rangle$ again.

More specifically, the state of the cavity-photon set-up after the mixing is

$$|\psi_{\rm enc}\rangle = \tfrac{1}{2}|gg\rangle(|ab\rangle + {\rm i}|bb\rangle + {\rm i}|aa\rangle - |ba\rangle) + \tfrac{1}{2}|gm\rangle(|ad\rangle + {\rm i}|ac\rangle + {\rm i}|bd\rangle - |bc\rangle) + \tfrac{1}{2}|mg\rangle(|cb\rangle + {\rm i}|ca\rangle + {\rm i}|db\rangle - |da\rangle) + \tfrac{1}{2}|mm\rangle(|cd\rangle + {\rm i}|cc\rangle + {\rm i}|dd\rangle - |dc\rangle), \tag{9}$$

where $|ij\rangle$ $(i, j = a, b, c, d)$ represent the state of the photons along the path i and j. Clearly, photonic states denoted by $|ii\rangle$ correspond to photon bunching in the respective detectors. However, a detection event in A and D respectively (or B and C) signifies that the atoms have collapsed to the state $1/\sqrt{2}(|gm\rangle - |mg\rangle)$, and a detection event in A and C (or B and D) means that the atoms have collapsed to the state $1/\sqrt{2}(|gm\rangle + |mg\rangle)$, otherwise the atoms are not maximally entangled and the system needs to be reset by initializing the atom to a linear superposed state of $|g\rangle$ and $|m\rangle$ again. Note that in this case, the original states of the atoms are destroyed unlike the previous scheme [11]. However, it would be interesting to investigate its possible realization using nanowires [15].

4. Inefficient detectors

Note that the implementation of such schemes does not necessarily require photon-number resolving detectors. Under ideal conditions, all outcomes of the photon pair measurement are distinguishable. However, in the real world, photon detectors have finite efficiency $\eta < 1$ so that photon generation succeeds only with a probability close to (but not equal to) 1 [16].

In figure 1, any failure to observe two photons in the set-up means that the atoms will be left in an unknown state. If such errors are small (less than 1%), the resulting gate failures can be overcome with existing fault tolerance techniques. Quantum computing with single photon sources with a high fidelity is nevertheless feasible, if one employs one-way quantum computing [17, 18] and uses photon detectors with a low-enough dark count rate.

By constructing a cluster state, it is possible under one-way quantum computing to realize an algorithm solely through local measurements, which can be performed with a high precision. To obtain a cluster state, one can use the RUS scheme to prepare an Ising gate [12] and apply the controlled phase operation wherever a cluster state bond is needed. For $\eta < 1$, one may not always know if the construction of a bond has succeeded or not; however if the procedure is not successful, one can always repeat the bonding attempt. Moreover, this construction can be done over an already prepared but smaller cluster state. As long as the efficiency of detectors and photon sources is not too low, an N-qubit cluster state can be built in a time faster than polynomial in N [18].

5. Conclusion

In this paper, we have reviewed and discussed at length the basic ideas behind RUS quantum computing scheme and suggested the possibility of using a different scheme for the purpose. We have also briefly discussed the possibility of constructing cluster states by repeatedly using entangling operations similar to the construction of maximally entangled stationary states.

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References

[1] Lütkenhaus N, Calsamiglia J and Suominen K A 1999 *Phys. Rev.* A **59** 3295
[2] Knill E, Laflamme R and Milburn G 2001 *Nature* **409** 46
[3] Gottesman D and Chuang I L 1999 *Nature* **402** 390
[4] Yoran N and Reznik B 2003 *Phys. Rev. Lett.* **91** 037903
Nielsen M A 2004 *Phys. Rev. Lett.* **93** 040503
Browne D E and Rudolph T 2005 *Phys. Rev. Lett.* **95** 010501
[5] Pittman T B, Jacobs B C and Franson J D 2002 *Phys. Rev. Lett.* **88** 257902
O'Brien J L, Pryde G J, White A G, Ralph T C and Branning D 2003 *Nature* **426** 264
[6] Schmidt-Kaler F, Häffner H, Riebe M, Gulde S, Lancaster G P T, Deuschle T, Becher C, Roos C F, Eschner J and Blatt R 2003 *Nature* **422** 408
[7] Leibfried D *et al* 2003 *Nature* **422** 412
[8] Protsenko I E, Reymond G, Schlosser N and Grangier P 2002 *Phys. Rev.* A **66** 062306
Schlosser N, Protsenko I and Grangier P 2003 *Phil. Trans. R. Soc.* A **361** 1527
[9] Duan L-M, Blinov B B, Moehring D L and Monroe C 2004 *Quant. Inform. Comp.* **4** 165
[10] Zou X B and Mathis W 2005 *Phys. Rev.* A **71** 042334
[11] Lim Y-L, Beige A and Kwek L-C 2005 *Phys. Rev. Lett.* **95** 030505 (see also *Preprint* quant-ph/0408043, 1–4)
[12] Lim Y-L, Barret S-D, Beige A, Kok P and Kwek L-C 2006 *Phys. Rev.* A **73** 012304
[13] Duan L M, Madsen M J, Moehring D L, Maunz P, Kohn R N Jr and Monroe C 2006 *Phys. Rev.* A **73** 062324
[14] Waks E and Vuckovic J 2006 *Phys. Rev. Lett.* **96** 153601
[15] Chang D E, Sørensen A S, Hemmer P R and Lukin M D 2005 *Preprint* quant-ph/0506117
Barrelet C J, Greytak A B and Lieber C M 2004 *Nanoletters* **4** 1981
[16] Kuhn A, Hennrich M and Rempe G 2002 *Phys. Rev. Lett.* **89** 067901
McKeever J, Boca A, Boozer A D, Miller R, Buck J R, Kuzmich A and Kimble H J 2004 *Science* **303** 1992
[17] Raussendorf R and Briegel H J 2001 *Phys. Rev. Lett.* **86** 5188
[18] Barrett S D and Kok P 2005 *Phys. Rev.* A **71** 060310